Chapter 7

1

ENERGY OF A SYSTEM

1

Lecture 05

2

Conservative Forces

2

Conservative Forces

3

The work done by a conservative force on a particle :

- Moving between any two points is independent of the path taken by the particle.
- Moving through any closed path is zero.
- Moving between any two points equals the decrease in potential energy. $W_{c.f} = -\Delta U$

Examples:

- Gravitational force
- Restoring force in a spring
- Electric force
- Magnetic force

3

Non-conservative Forces

4

A non-conservative force does not satisfy the conditions of conservative forces.

Non-conservative forces acting in a system cause a *change* in the mechanical energy of the system.

$$E_{mech.} = K + U$$

K includes the kinetic energy of all moving members of the system.

U includes all types of potential energy in the system.

4

Conservative Forces and Potential Energy

5

Define a potential energy function, U, such that the work done by a conservative force equals the decrease in the potential energy of the system.

The work done by such a force, F, is

$$W_{c.f} = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

ΔU is negative when F and x are in the same direction.

5

Conservative Forces and Potential Energy

6

The conservative force is related to the potential energy function through.

$$F_x = -\frac{dU}{dx}$$

- The x component of a conservative force acting on an object within a system equals the negative of the potential energy of the system with respect to x.
- Can be extended to three dimensions

$$F_x = -\frac{\partial U}{\partial x} \qquad F_y = -\frac{\partial U}{\partial y} \qquad F_z = -\frac{\partial U}{\partial z}$$

6

Conservative F and U - 1

Saturday, 30 January, 2021 15:13

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.

H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A potential energy function for a system in which a three-dimensional force acts is of the form $U(x, y, z) = 3x^3y - 7xz$. Find the force that acts at the point (x, y, z).

$$F_x = -\frac{\partial U}{\partial x} = -\frac{\partial}{\partial x}(3x^3y - 7xz) = -(9x^2y - 7z)$$

$$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial}{\partial y}(3x^3y - 7xz) = -(3x^3 - 0)$$

$$F_z = -\frac{\partial U}{\partial z} = -\frac{\partial}{\partial z}(3x^3y - 7xz) = -(0 - 7x)$$

$$\vec{F} = -(9x^2y - 7z)\hat{\imath} - 3x^3\hat{\jmath} + 7x\hat{k}$$

Conservative F and U - 2

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A small block with mass $0.04\ kg$ is moving in the xy-plane. The net force on the block is described by the potential energy function $U(x,y) = 15.8x^2 - 13.6y^3$. What are the magnitude and direction of the acceleration of the block when it is at the point $(x = 0.3\ m, y = 0.6\ m)$?

$$F_x = -\frac{\partial U}{\partial x} = -\frac{\partial}{\partial x}(15.8x^2 - 13.6y^3) = -31.6x$$

$$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial}{\partial y}(15.8x^2 - 13.6y^3) = -40.8y^2$$

$$\vec{F} = -31.6x\hat{\imath} - 40.8y^2\hat{\jmath}$$

$$\vec{F}_{\substack{x=0.3\\y=0.6}} = -31.6 \times 0.3\hat{\imath} - 40.8 \times 0.6^2\hat{\jmath}$$

$$\vec{F} = -94.8\hat{\imath} - 14.7\hat{\jmath}\ (N)$$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{-94.8\hat{\imath} - 14.7\hat{\jmath}}{0.04} = -2370\hat{\imath} - 367.5\hat{\jmath}\ (m/s^2)$$

The magnitude of $\vec{a}$:

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-2370)^2 + (-367.5)^2} = 2398\ m/s^2$$

The direction of $\vec{R}$:

$$\Rightarrow\ \theta = tan^{-1}\left(\frac{-367.5}{-2370}\right) = 8.8°$$

Since the sign of a_x is negative and the sign of a_y is negative, the resultant acceleration lies in the third quadrant of the coordinate system. That is,

$$\theta = 180° + 8.8° = 188.8°$$

Conservative F and U - 3

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A single conservative force acts on a $5\ kg$ particle within a system due to its interaction with the rest of the system. The equation $F_x = 2x + 4$ describes the force, where F_x is in Newtons and x is in meters. As the particle moves along the x axis from $x = 1\,m$ to $x = 5\,m$, calculate:

- the work done by this force on the particle,
- the change in the potential energy of the system, and
- the kinetic energy the particle has at $x = 5\,m$ if its speed is $3\ m/s$ at $x = 1\,m$.

Solution

$$W = \int_1^5 (2x + 4)\,\mathrm{d}x = 40J$$

Solution

$$W = -\Delta U \rightarrow \Delta U = -W = -40J$$

Solution

$$W = \Delta K = K_f - K_i$$

$$K_f = W + K_i = W + \frac{1}{2}mv_i^2$$

$$K_f = 40 + \left(\frac{1}{2} \times 5 \times 3^2\right) = 62.5J$$